Ottimizzazione di Forma di Strutture a Telaio attraverso un approccio misto Analitico-2D e Numerico-3D

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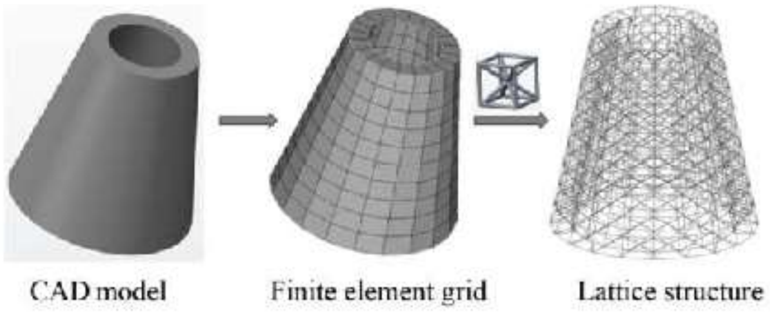
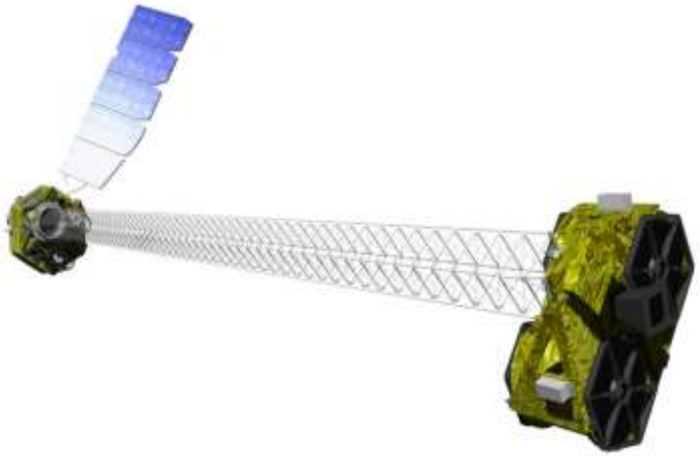
Structure

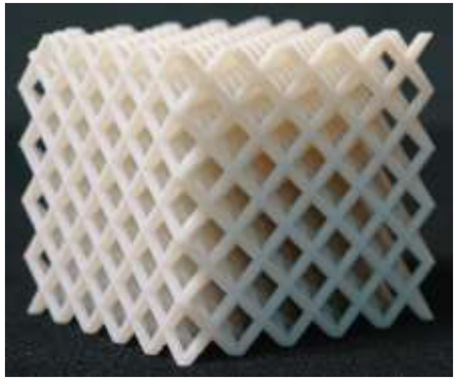
- Introduction
- Methods
- Application Test Cases

Introduction

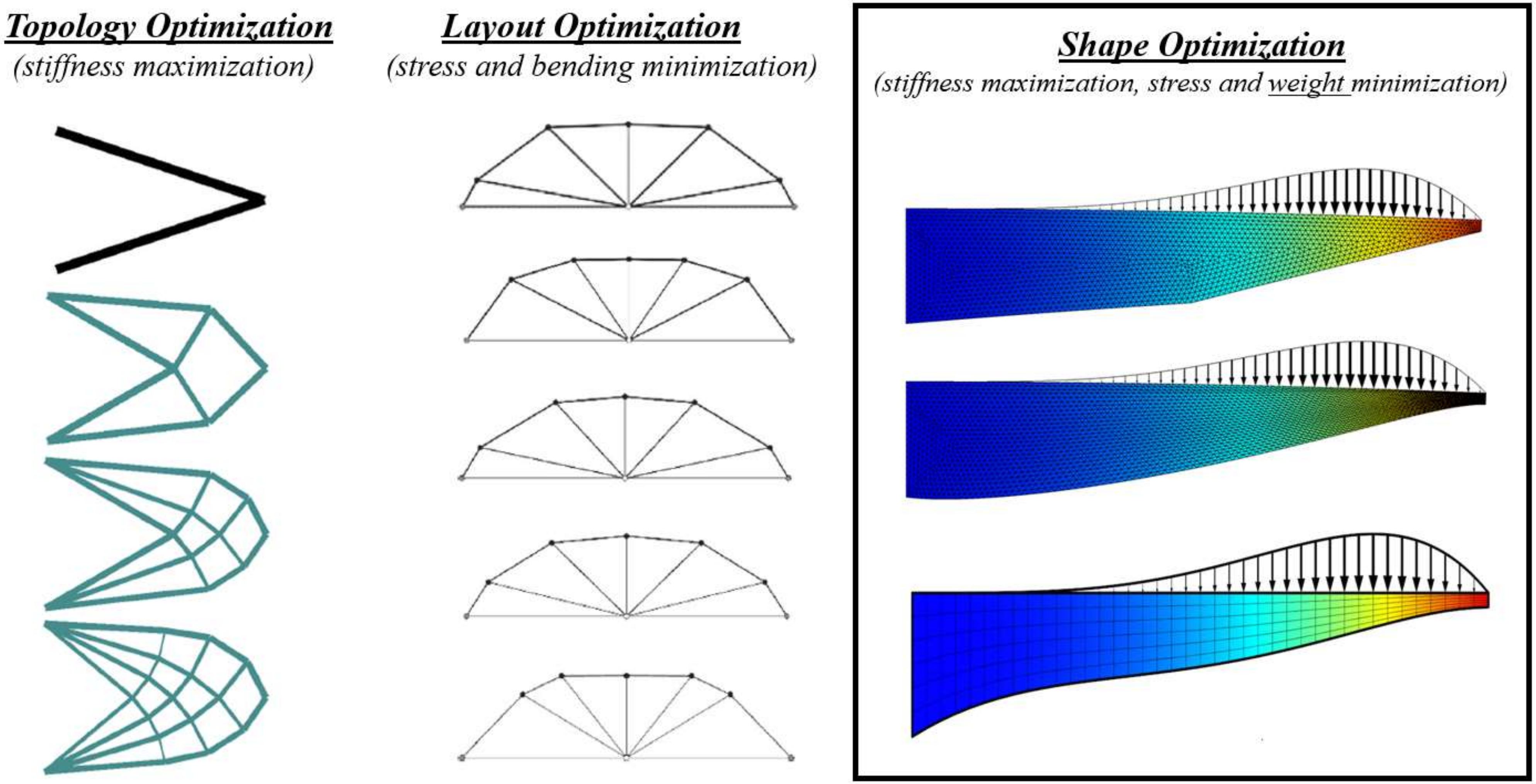
* The Bayonne Bridge (FR)







Introduction



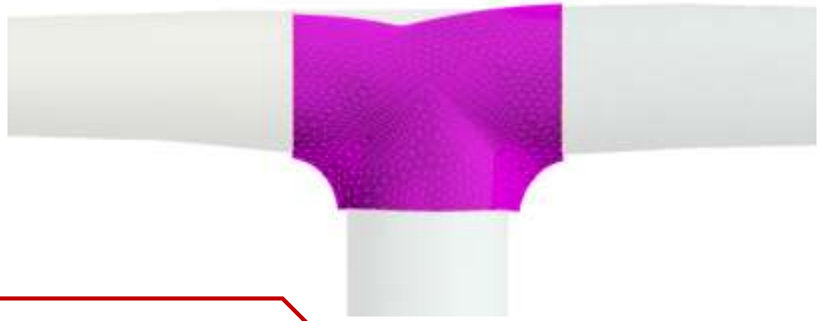
Introcuction

- Classification of Optimization Methods:
  - Zero-Order Methods: DOE-based, Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Biological Growth Method (BGM)
  - Gradient-Based Methods: Penalty methods, Sequential Quadratic Programming (SQP), Trust Region, Damped Least-Squares (DLS, also known as the Levenberg-Marquardt algorithm), Inscribed Hyperspheres (IHS), Gradient Projection Method
  - Analytical and Numerical Methods

- The development of additive manufacturing and homogenization techniques has made this topic even more interesting and relevant.

Methods

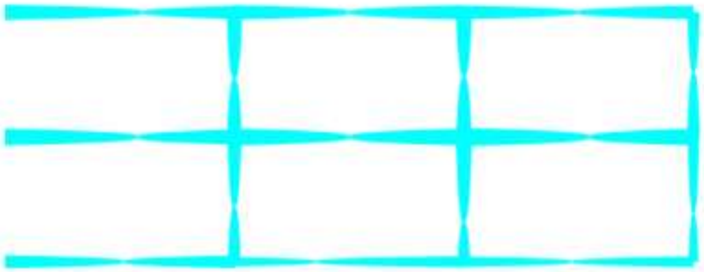
- Workflow



Step 1: Analytical Optimization - Uniform Strength

Creation of FEM Model with Mesh Morphing

Step2: Numerical Optimization - BGM



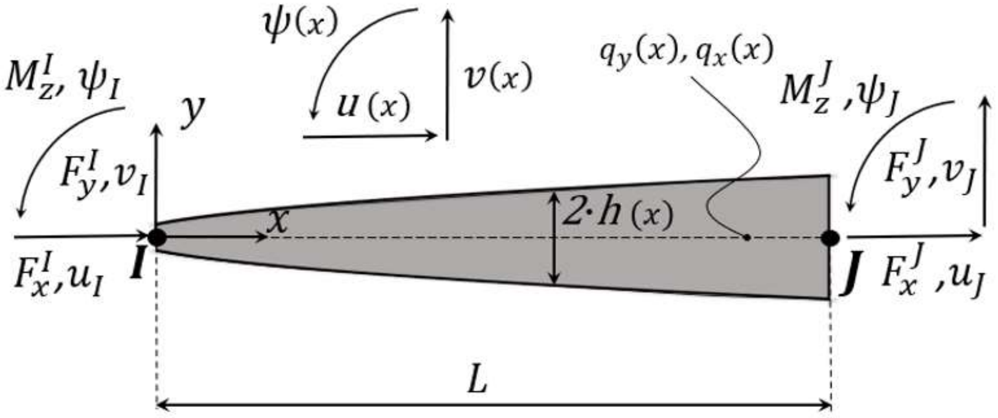
Methods

- Step 1: Analytical Optimization - Uniform Strength (1/2)

Equilibrium

$$\begin{cases} N(x) = -F_x^I - \int_{x^*}^{x} q_x(\tilde{x})\, d\tilde{x} \\ T(x) = -F_y^I - \int_{x^*}^{x} q_y(\tilde{x})\, d\tilde{x} \\ M(x) = -M_z^I + x \cdot F_y^I + \int_{x^*}^{x} q_y(\tilde{x})\, (x - \tilde{x}) d\tilde{x} \end{cases}$$

$$\begin{cases} F_x^J = -F_x^I - \int_{x^*}^{L} q_x(\tilde{x})\, d\tilde{x} \\ F_y^J = -F_y^I - \int_{x^*}^{L} q_y(\tilde{x})\, d\tilde{x} \\ M_z^J = -M_z^I + L \cdot F_y^I + \int_{x^*}^{L} q_y(\tilde{x})\, (L - \tilde{x}) d\tilde{x} \end{cases}$$



Kinematic

$$\begin{cases} u'(x) = \dfrac{1}{EA(x)} \left[-F_x^I - \int_{x^*}^{L} q_x(\tilde{x})\, d\tilde{x} \right] \\ \psi'(x) = \dfrac{1}{EI(x)} \left[-M_z^I + x \cdot F_y^I + \int_{x^*}^{x} q_y(\tilde{x})\, (x - \tilde{x}) d\tilde{x} \right] \\ v'(x) = \psi \end{cases}$$

Methods

- Step 1: Analytical Optimization - Uniform Strength (2/3)

Why the Uniform Strength? → *Maximum mechanical efficiency*

$$\mathscr{L} = \frac{\text{Elastic Strain Energy}}{\text{maximum energy that can be accumulated}} = \frac{1}{(\sigma_{max})^2\, V} \int_L \frac{M^2(x)}{I(x)}\, dx =$$

→ $\frac{1}{9}$ *for constant section*

→ $\frac{1}{3}$ *for Uniform Strength beams (variable sections)*

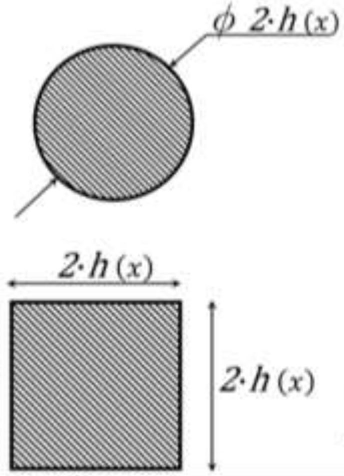
How to obtain the cross-section shape to guarantee the Uniform Strength?

$$\sigma_x(x,y) = \frac{N(x)}{A(x)} - \frac{M(x)}{I(x)}\, y$$

$$max\big(\sigma_x(x, \pm h(x)/2)\big) = \sigma_{iso} = const \longrightarrow \sigma_{iso} = \frac{|N(x)|}{A(x)} + \frac{|M(x)|}{I(x)} \frac{h(x)}{2}$$

Methods

- Step 1: Analytical Optimization - Uniform Strength (3/3)



$$h(F_x^I, F_y^I, M_z^I, x) = \sqrt[3]{\frac{q}{2}}\left[\sqrt[3]{-1+\sqrt{\Delta}} + \sqrt[3]{-1-\sqrt{\Delta}}\right]$$

$$\Delta = 1 + \frac{4}{27}\frac{p^3}{q^2} \quad ; \quad p(F_x^I, x) = -\frac{|N(x)|}{c_A\ \sigma_{iso}} \quad ; \quad q(F_y^I, M_z^I, x) = -\frac{|M(x)|}{c_I\ \sigma_{iso}}$$

Circular cross-section

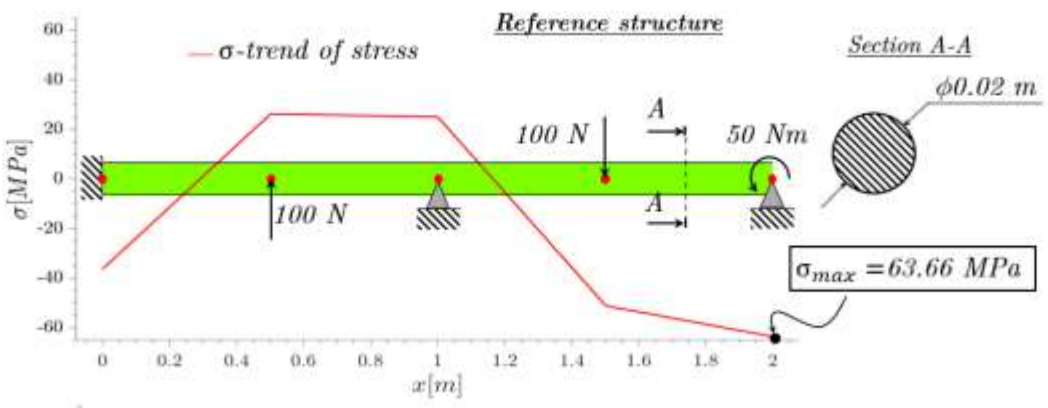
$c_A = \pi \quad , \quad c_I = \pi/4$

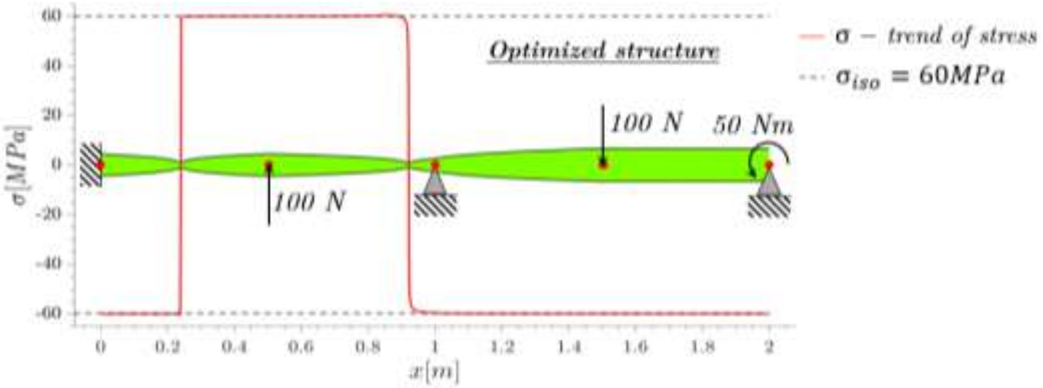
Square cross-section

$c_A = 1 \quad , \quad c_I = 1/12$

If the stress contributes due to normal force is neglected and the distributed loads are nulls, it is possible to integrate the kinematic equations:

$$h(F_y^I, M_z^I, x) \cong \left[c_M\ s(x) \cdot M(F_y^I, M_z^I, x)\right]^{\frac{1}{3}}$$





Methods

- Radial Basisi Functions (RBF) Mesh Morphing

$$f^x(x) = \sum_{i=1}^{m} \gamma_i^x \phi(\|c_i - x\|) + \beta_1^x + \beta_2^x x_1 + \beta_3^x x_2 + \beta_4^x x_3$$
$$f^y(x) = \sum_{i=1}^{m} \gamma_i^y \phi(\|c_i - x\|) + \beta_1^y + \beta_2^y x_1 + \beta_3^y x_2 + \beta_4^y x_3$$
$$f^z(x) = \sum_{i=1}^{m} \gamma_i^z \phi(\|c_i - x\|) + \beta_1^z + \beta_2^z x_1 + \beta_3^z x_2 + \beta_4^z x_3$$

$$\begin{bmatrix} M & P \\ P^T & 0 \end{bmatrix} \begin{Bmatrix} \gamma \\ \beta \end{Bmatrix} = \begin{Bmatrix} g \\ 0 \end{Bmatrix}$$

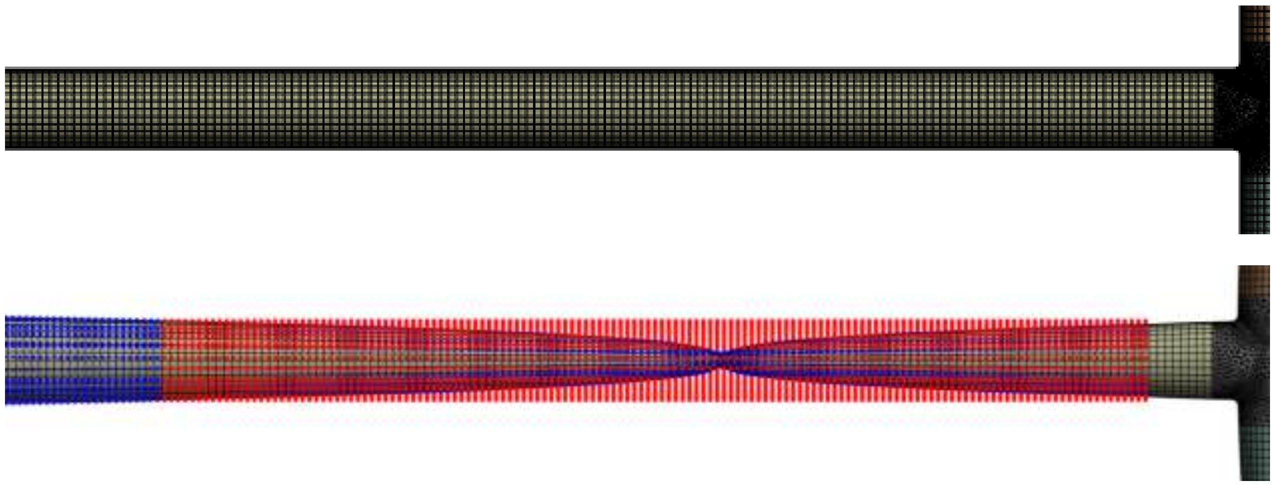
$With \quad M = \phi(\|c_i - c_j\|)$

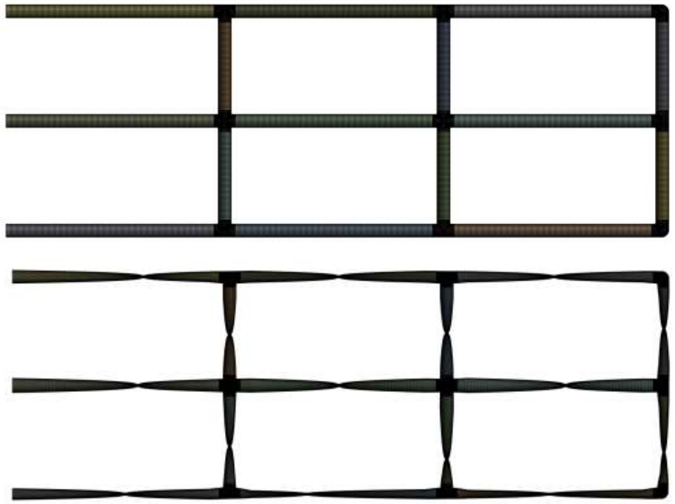
$P_j = [1 \;\; x_1 \;\; x_2 \;\; \dots \;\; x_n]$

Weight and radial function

Polynomial term

Boundary conditions

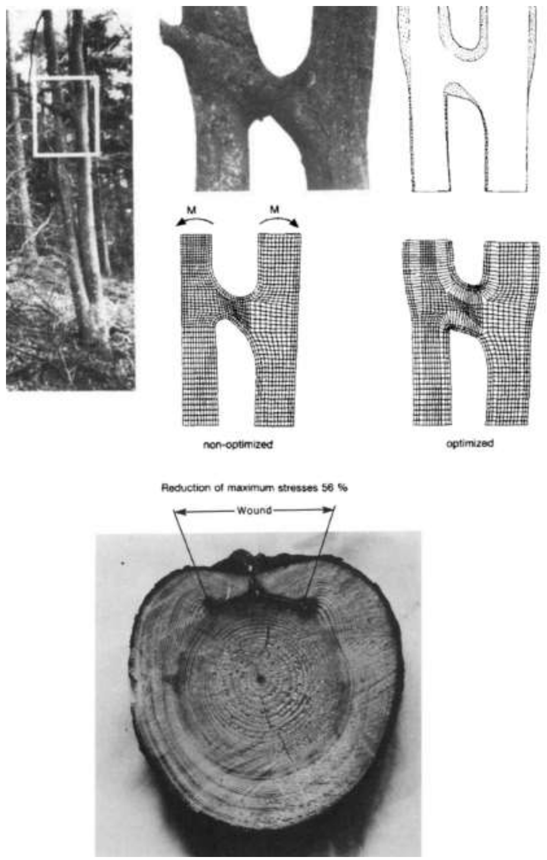




Methods

- Step 2: Numerical Optimization - BGM (1/2)
  - BGM approach is based on the observation that biological structures growth is driven by local level of stress.
  - Bones and trees' trunks are able to adapt the shape to mitigate the stress level due to external loads.
  - The process is driven by stress value at surfaces. Material can be added or removed according to local values.
  - Was proposed by Mattheck & Burkhardt in 1990.
  - The BGM idea is that surface growth can be expressed as a linear law with respect to a given threshold value:

$$\dot{\varepsilon} = k\left(\sigma_{Mises} - \sigma_{ref}\right)$$



Methods

- Step 2: Numerical Optimization - BGM (2/2)
  - Different stress types can be used to modify the surface shape.

Stress/strain type	Equation	Stress/strain type	Equation
von Mises stress	$\sigma_e = \sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$	Stress intensity	$\sigma_e = max(\lvert\sigma_1-\sigma_2\rvert, \lvert\sigma_2-\sigma_3\rvert, \lvert\sigma_3-\sigma_1\rvert)$
Maximum principal stress	$\sigma_e = max(\sigma_1, \sigma_2, \sigma_3)$	Maximum Shear stress	$\sigma_e = 0.5 \cdot (max(\sigma_1, \sigma_2, \sigma_3) - min(\sigma_1, \sigma_2, \sigma_3))$
Minimum principal stress	$\sigma_e = min(\sigma_1, \sigma_2, \sigma_3)$	Eqv. plastic strain	$\varepsilon_e = [2(1+\nu')]^{-1} \cdot \left(0.5\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}\right)$

  - Automatic optimization is accoplished connecting adjoint and BGM data from numerical simulation to mesh morphing tool.
  - Offset Surface shape modification allow to define for each node a displacement according to the local normal direction.
  - When using BGM data, the direction and amplitude of displacement is defined according to BGM stress data, considering the threshold stress value σth and the d maximum displacement:

$$S_{node} = \frac{\sigma_{node} - \sigma_{th}}{\sigma_{max} - \sigma_{min}} \cdot d$$

Methods

- Performance Factors:

  - Surface exploitation factor: $f_1 = \frac{\sqrt{\sum_{i=1}^{n_S}(\sigma_{ISO} - \sigma_{VM})^2}}{n_S} \frac{1}{\sigma_{ISO}}$

  - Volume exploitation factor: $f_2 = \frac{\sqrt{\sum_{i=1}^{n_V}(\sigma_{ISO} - \sigma_{VM})^2}}{n_V} \frac{1}{\sigma_{ISO}}$

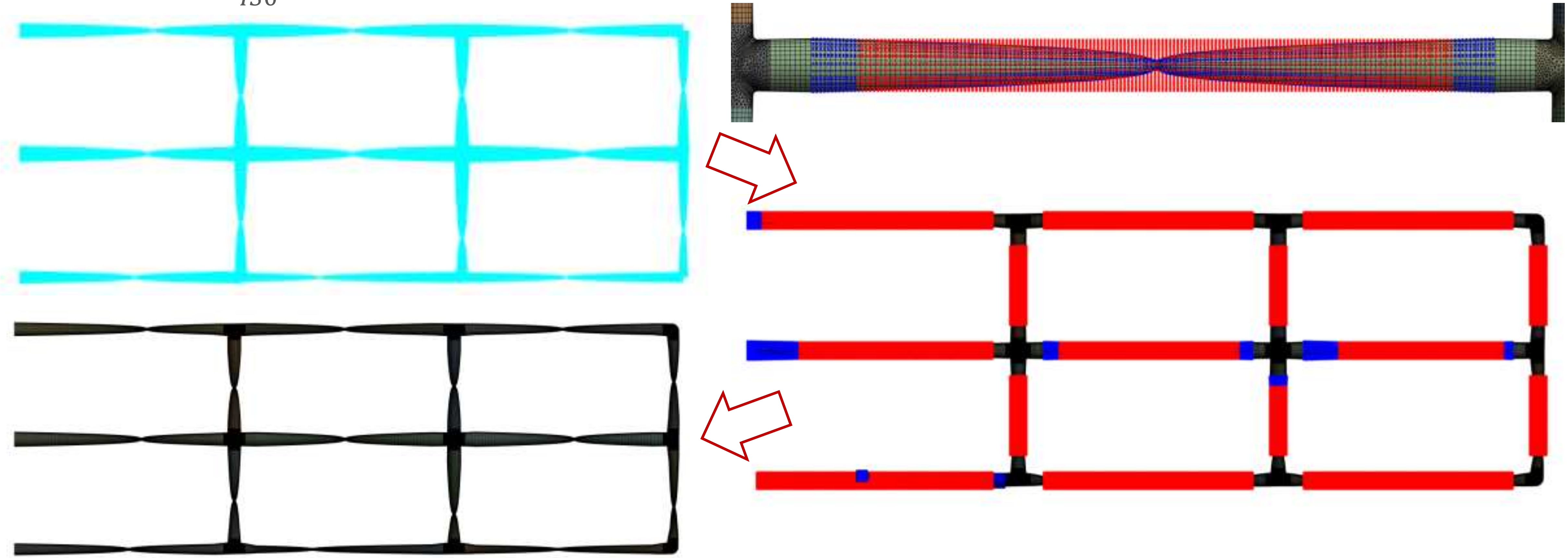
  - Energy factor : $f_3 = \frac{E_d * 2E}{{\sigma_{ISO}}^2 V}$

Application Test Cases

- Baseline Testcase 1
- Fixed constraints
- F = 7kN

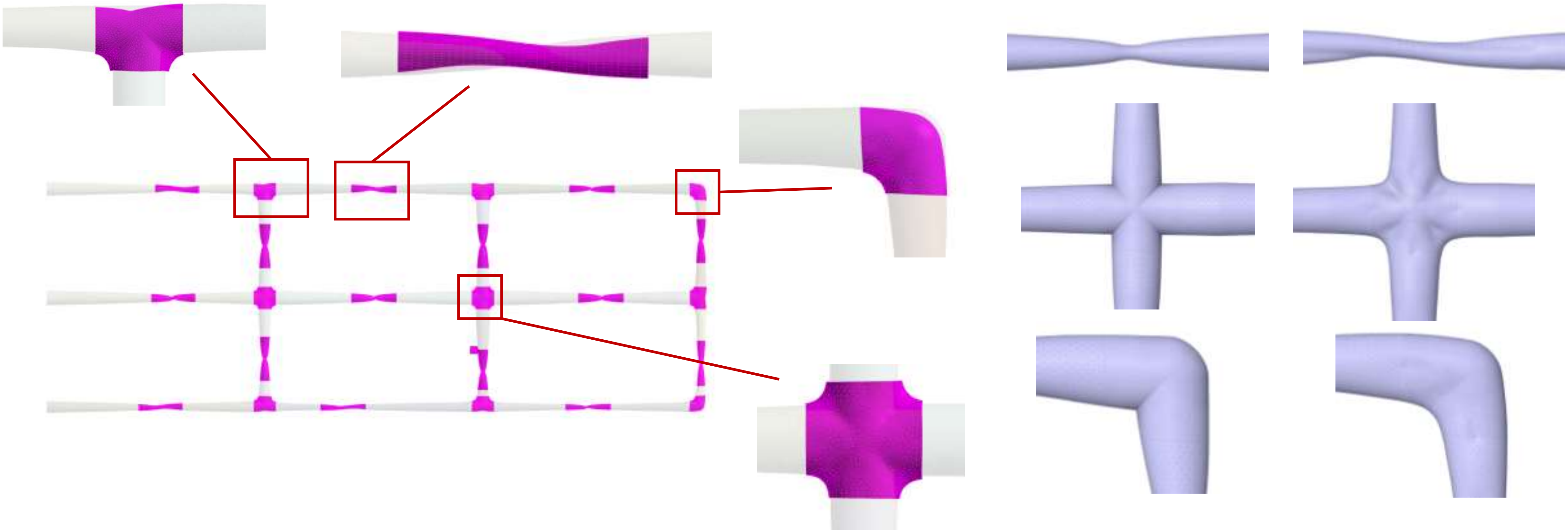
Application Test Cases

- Testcase 1: Analytical Optimization
  - $\sigma_{IS0} = 2.5e^{8}$ Pa



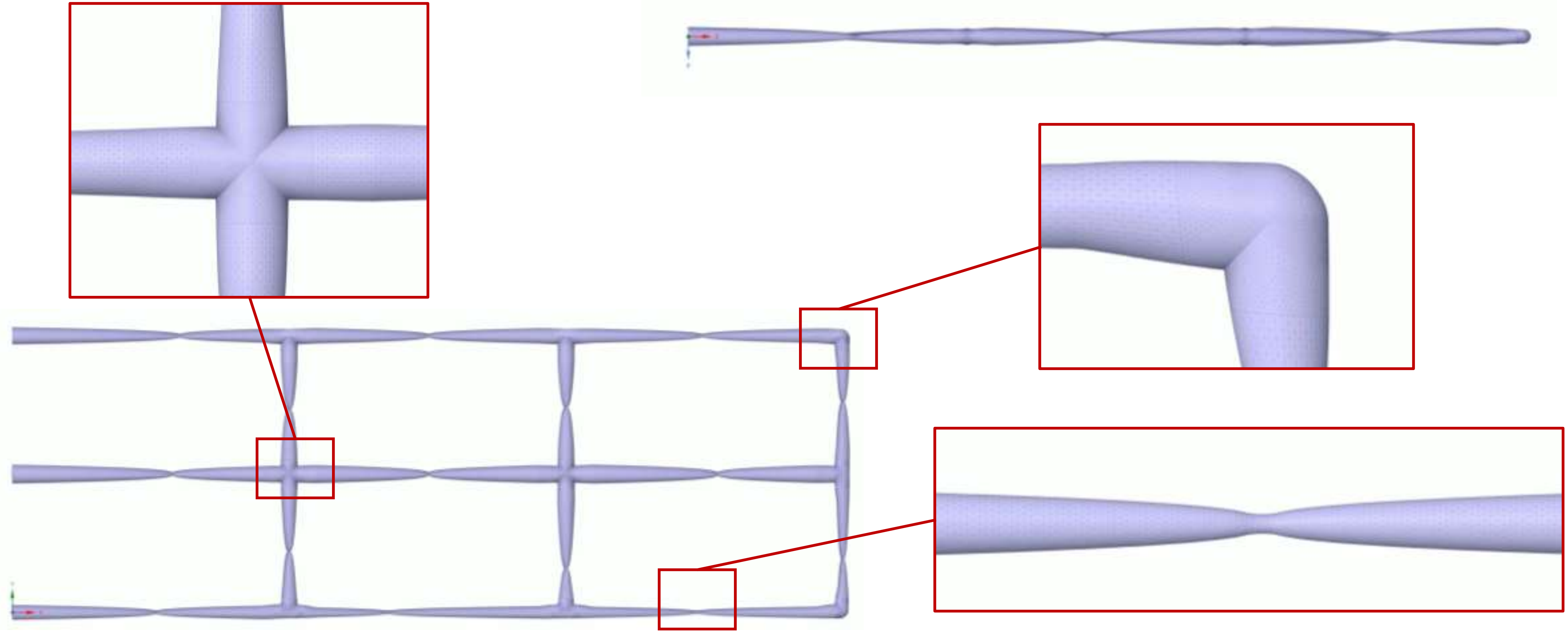
Application Test Cases

- Testcase 1: BGM (1/2)



Application Test Cases

- Testcase 1: BGM (2/2)



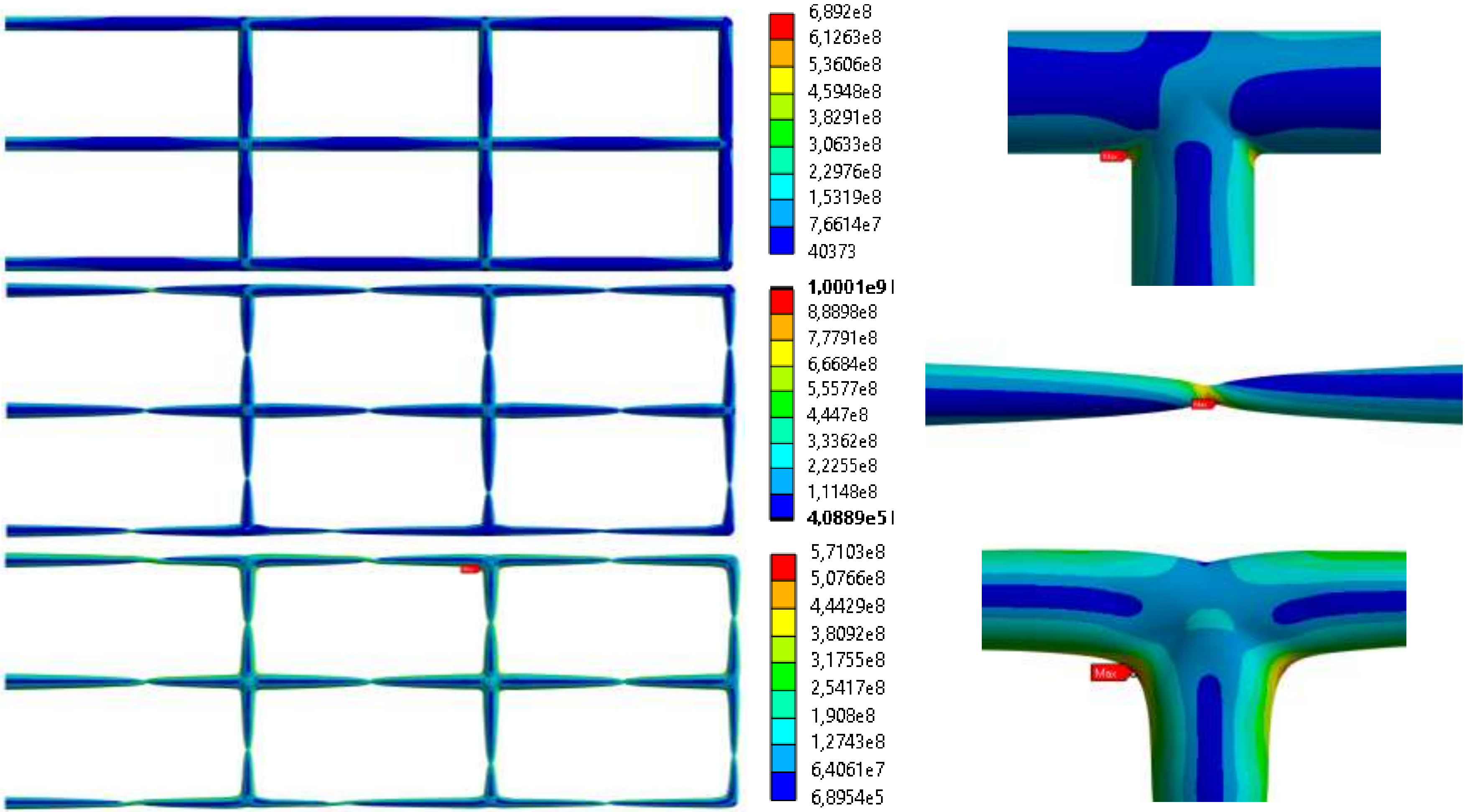
Application Test Cases

- Testcase 1: VM stress

Baseline

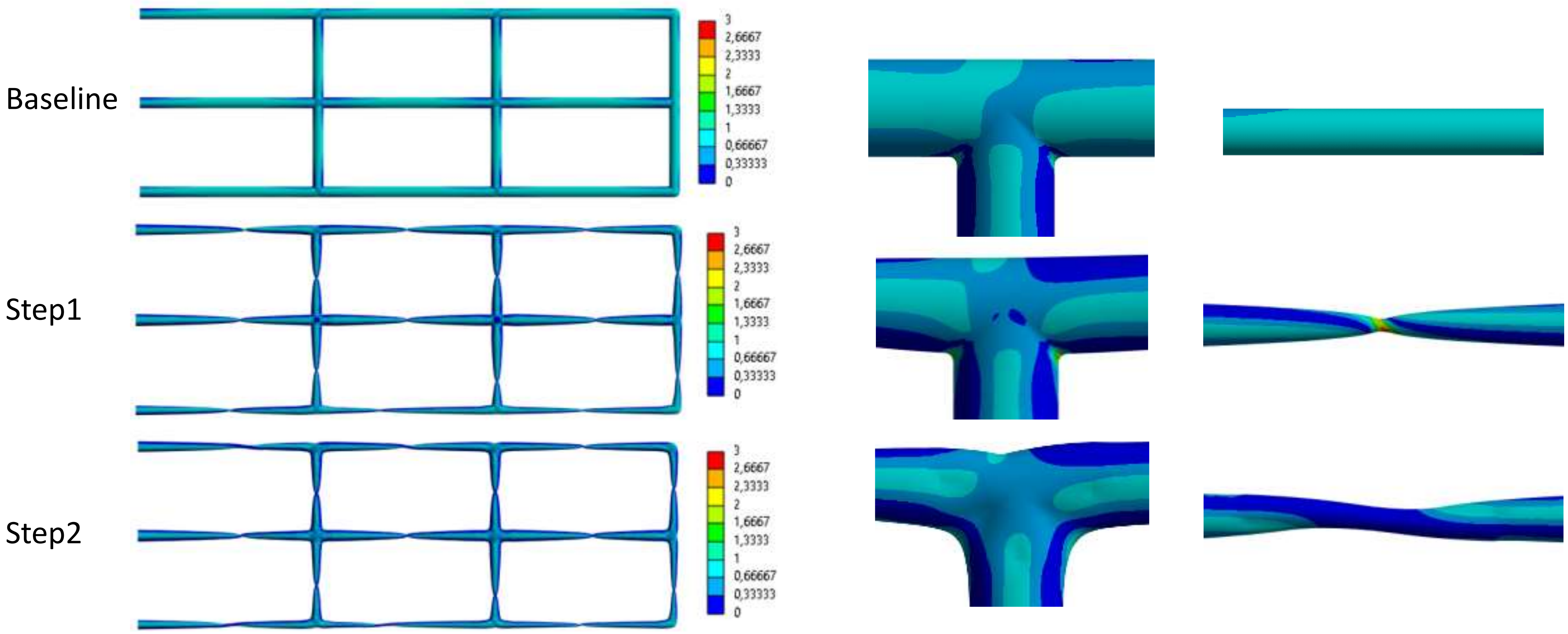
Step1

Step2



Application Test Cases

- Testcase 1: Surface exploitation factor



Application Test Cases

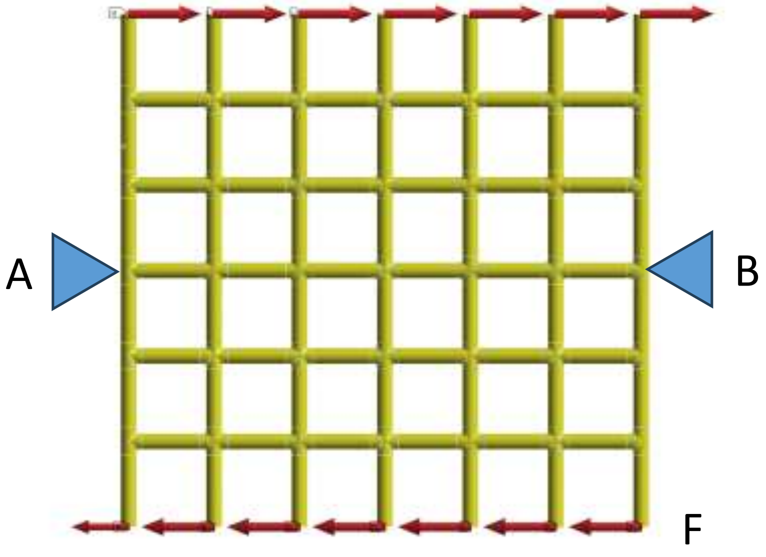
- Testcase 1: Results Comparison

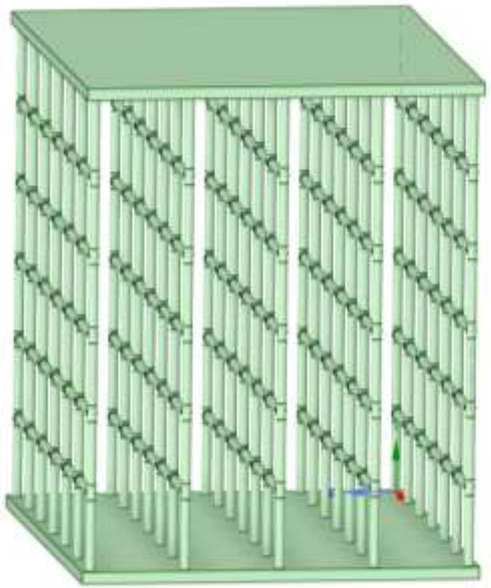
	VM_max [Pa]	Vol [m^3]	Surface exploitation factor	Volume exploitation factor
Baseline	6,9e8	4,16e-003	0,72	0,8
Step1	1e9	2,24 e-003	0,53	0,64
Step2	5,7e8	1,95 e-003	0,48	0,612

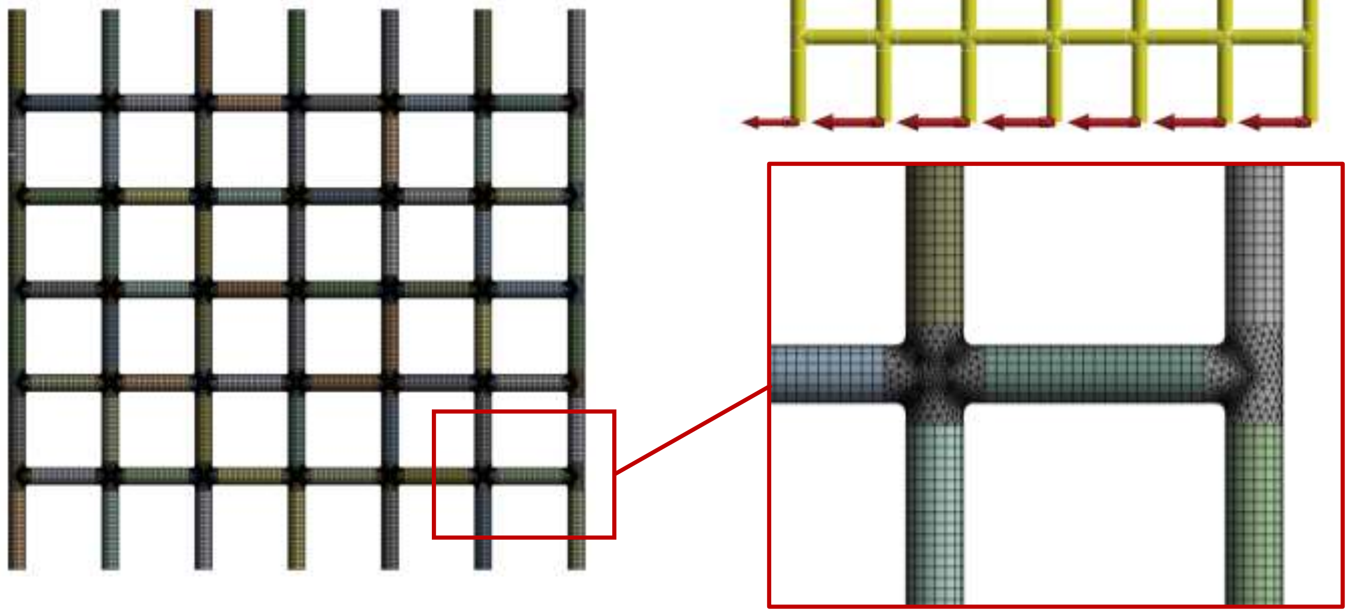
	Strain energy	Energy factor
Baseline	48,221 J	0,075
Step1	98,108 J	0,28
Step2	102,23 J	0,335

Application Test Cases

- Testcase 2: Baseline
  - Y shift and rotations locked at the top and bottom
  - Z-shift locked
  - Movement blocked in A and B
  - F = 35 N

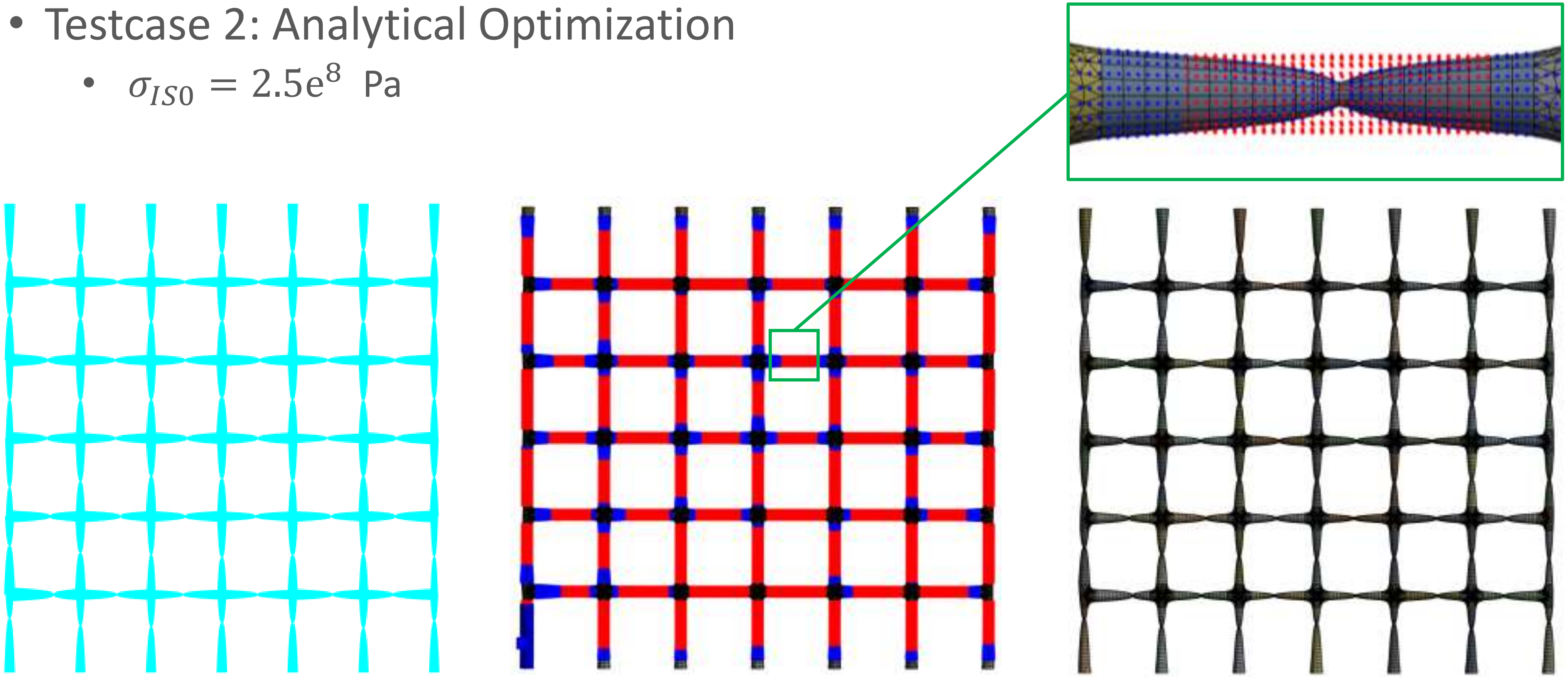






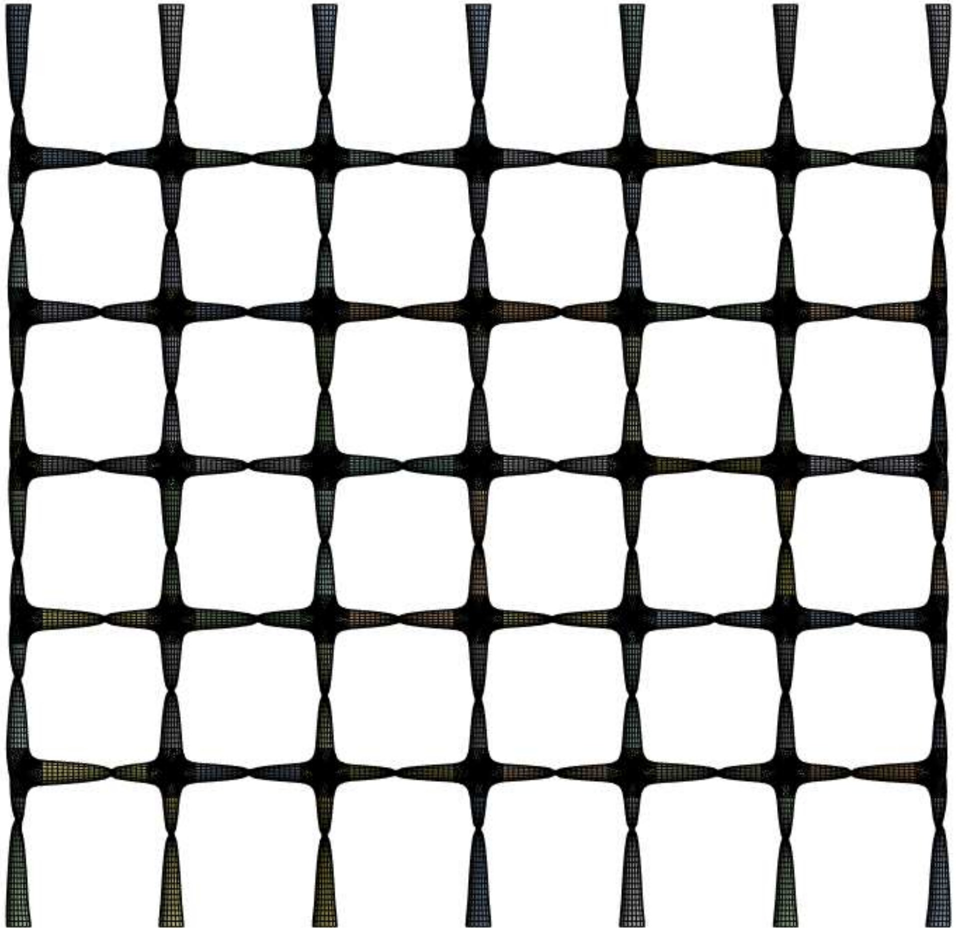
Application Test Cases

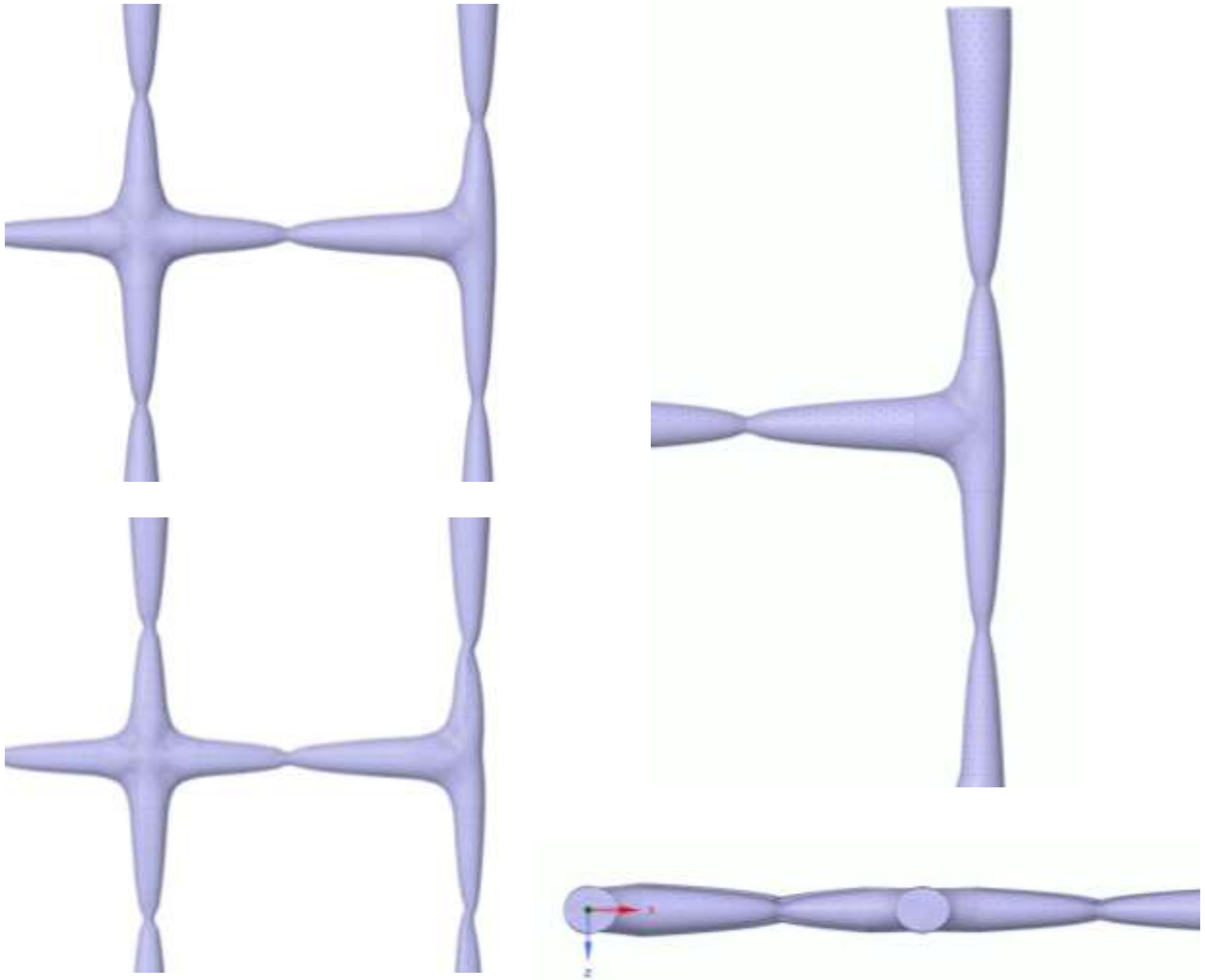
- Testcase 2: Analytical Optimization
  - $\sigma_{IS0} = 2.5e^{8}$ Pa



Application Test Cases

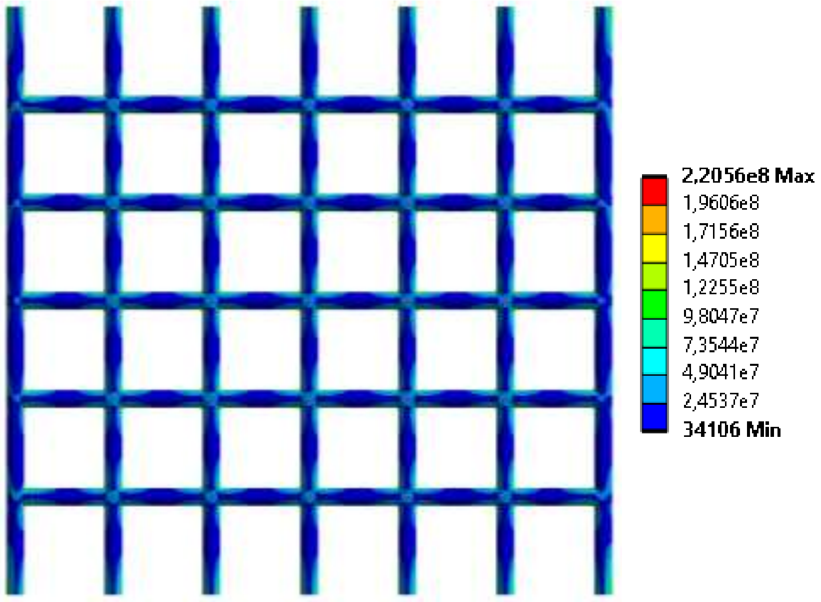
- Testcase 2: BGM

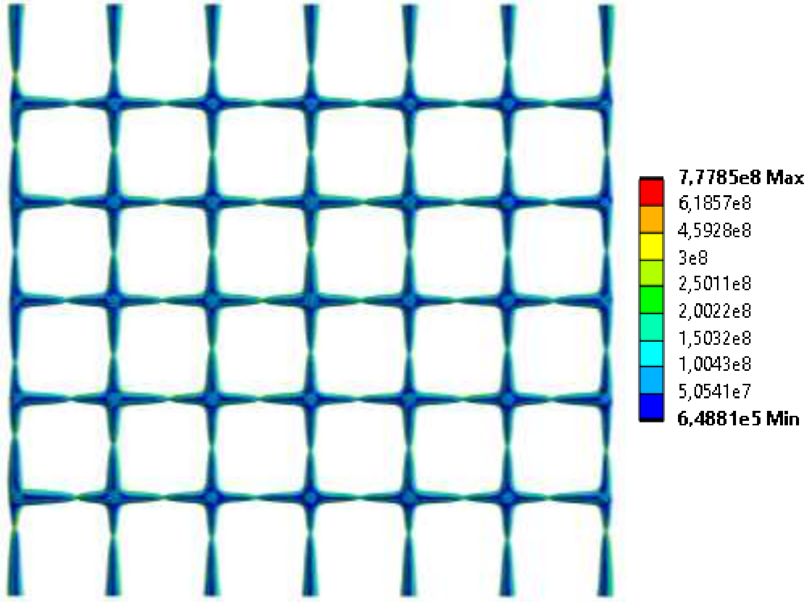


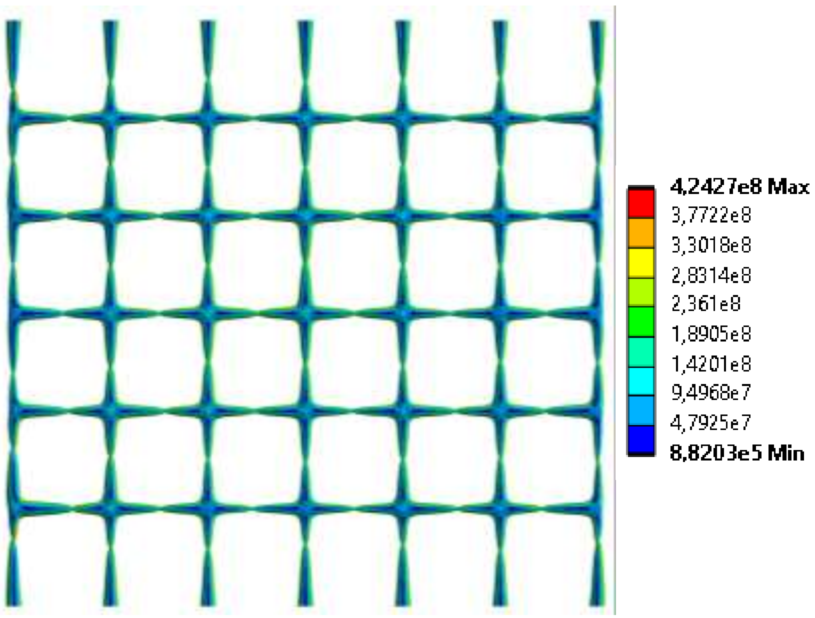


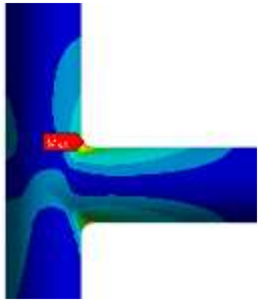
Application Test Cases

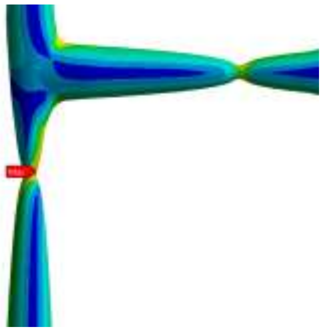
- Testcase 2: VM stress

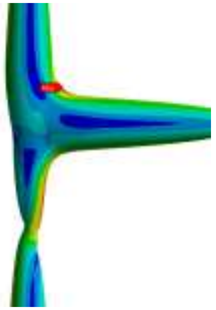








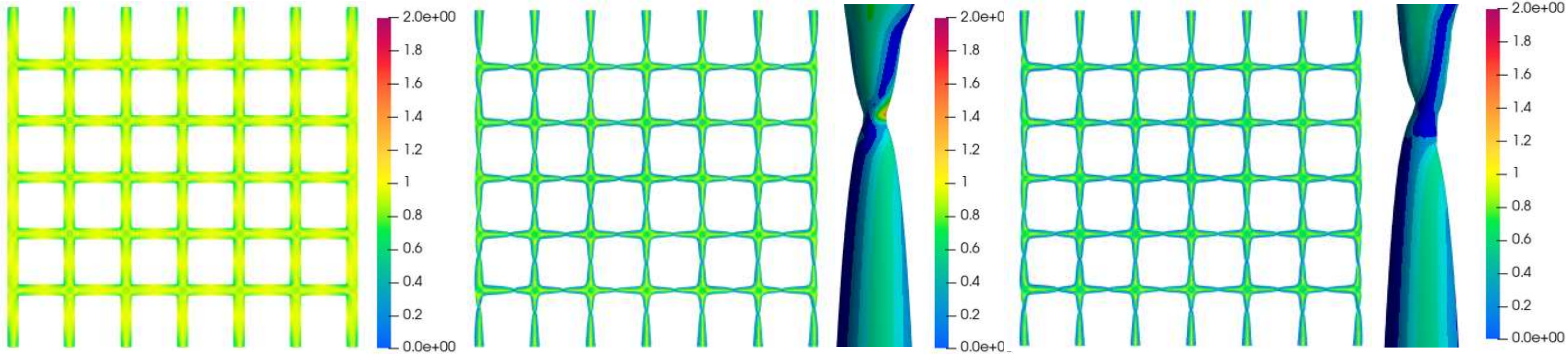




Application Test Cases

- Testcase 2: Surface exploitation factor

Baseline Step1 Step2



Application Test Cases

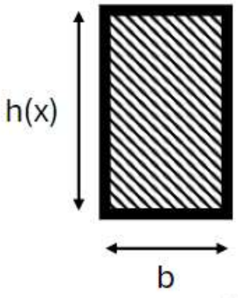
- Testcase 2: Results Comparison

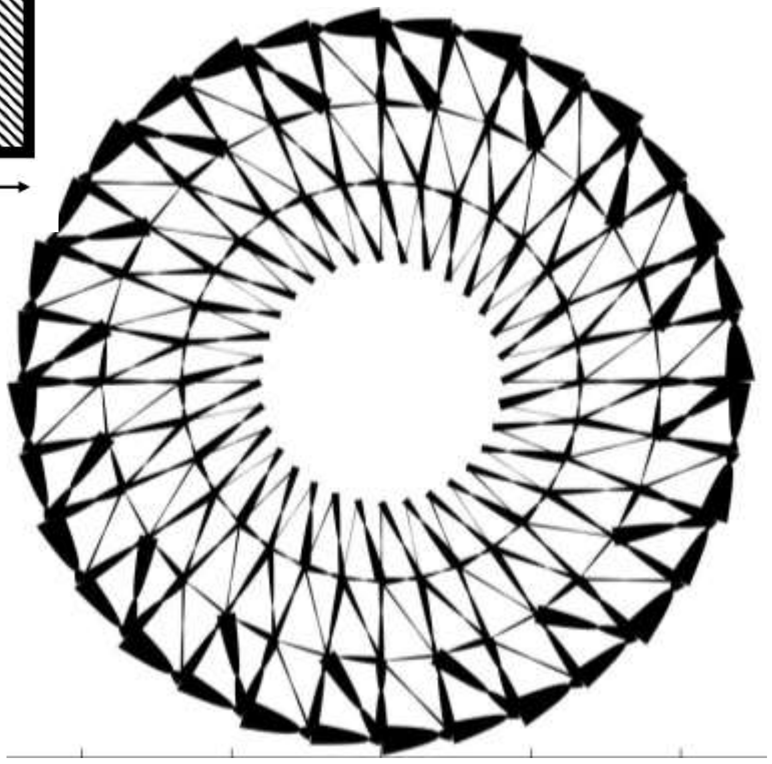
	VM_max [Pa]	Vol [m^3]	Surface exploitation factor	Volume exploitation factor
Baseline	2,2e8	7,9553e-006	0,85	0,87
Step1	7,8e8	2,8534e-006	0,6	0,69
Step2	4,2e8	2,3051e-006	0,55	0,64

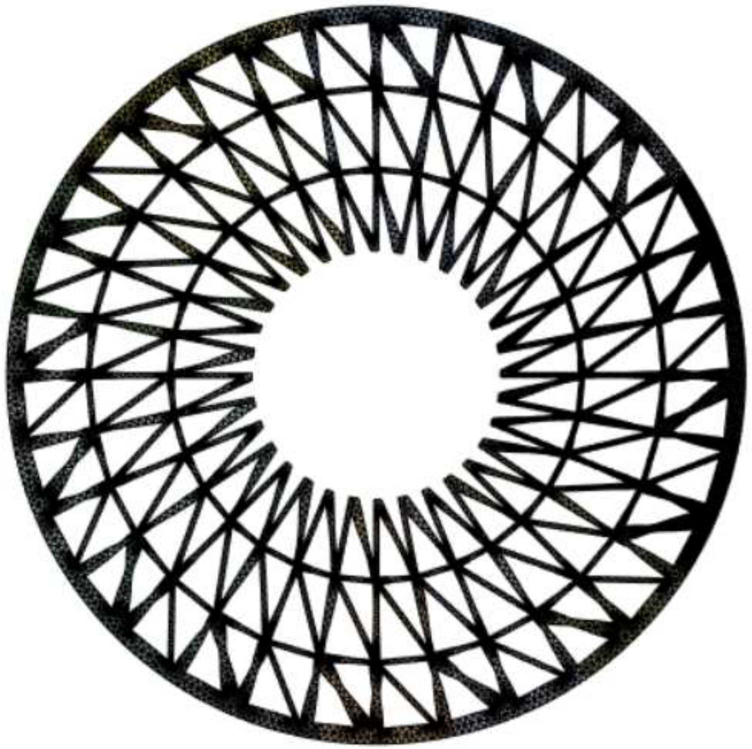
	Strain energy	Energy factor
Baseline	1,8075e-002 J	0,014
Step1	9,1444e-002 J	0,2
Step2	0,10943 J	0,3

Application Test Cases

- Testcase 3







Conclusions

- Next Steps:
  - Application of the workflow to an industrial case of interest
  - Finalization of the workflow with additive manufacturing to demonstrate the full success of the method

- Conclusions:
  - The tested workflow proves to be very effective, significantly improving material usage by optimizing the shape of the considered structures
  - The workflow can also be scaled for industrial problems of interest
  - The entire process can be automated, creating a design tool for frame structures