Automatic shape optimization of structural components with manufacturing constraints

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Outline

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• BGM Background
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Introduction

• Mechanical component optimization is a paramount target in every engineering application.

• A valuable tool for optimization in complex load and constraint configuration is the Finite Element Method (FEM), which allows to test different configurations before the prototyping phase.

• Optimization strategies are often based on parametrization of the FEM model: the optimal configuration is found among a family of configurations obtained varying the parameters describing the model geometry.

• Another possible optimization strategy exploits the results coming from FEM: Biological Growth Method (BGM) derives the component shape modification analysing the surface stress levels.
Introduction

• Both procedures, parameter based and BGM, require the generation of additional FEM models: this task can be very time-consuming specially dealing with complex shape components.

• To overcome this problem Mesh morphing can be adopted: it allows to generate new FEM models without modifying the geometry and without remesh it.

• Furthermore, in conjunction with the BGM approach, thanks to mesh morphing a high grade of automation can be achieved.

• In the present work, the tool adopted for morphing the FEM mesh is RBF Morph™, which is based on Radial Basis Functions (RBFs).
RBF Background

• RBFs are a mathematical tool capable to **interpolate** in a generic point in the space a function **known** in a discrete set of points (**source points**).

• The interpolating function is composed by a **radial basis** and by a **polynomial**:

\[
s(x) = \sum_{i=1}^{N} \gamma_i \varphi \left( \| x - x_{ki} \| \right) + h(x)
\]

- **radial basis**: distance from the i-th source point
- **polynomial**: graph
RBF Background

• If evaluated on the source points, the interpolating function gives exactly the input values:

\[ s(x_{k_i}) = g_i \]
\[ h(x_{k_i}) = 0 \quad 1 \leq i \leq N \]

• The RBF problem (evaluation of coefficients \( \gamma \) and \( \beta \)) is associated to the solution of the linear system, in which \( M \) is the interpolation matrix, \( P \) is a constraint matrix, \( g \) is the vector of known values on the source points:

\[
\begin{bmatrix}
M & P \\
P^T & 0
\end{bmatrix}
\begin{bmatrix}
\gamma \\
\beta
\end{bmatrix}
= 
\begin{bmatrix}
g \\
0
\end{bmatrix}
\]

\[ M_{ij} = \varphi(x_{k_i} - x_{k_j}) \quad 1 \leq i, j \leq N \quad P =
\begin{bmatrix}
1 & x_{k_1} & y_{k_1} & z_{k_1} \\
1 & x_{k_2} & y_{k_2} & z_{k_2} \\
M & M & M & M \\
1 & x_{k_N} & y_{k_N} & z_{k_N}
\end{bmatrix}
\]
RBF Background

- Once solved the RBF problem each displacement component is interpolated:

\[
\begin{align*}
    s_x(x) &= \sum_{i=1}^{N} \gamma_i \varphi(\|x-x_i\|) + \beta_1^i + \beta_2^i x + \beta_3^i y + \beta_4^i z \\
    s_y(x) &= \sum_{i=1}^{N} \gamma_i \varphi(\|x-x_i\|) + \beta_1^i + \beta_2^i y + \beta_3^i x + \beta_4^i z \\
    s_z(x) &= \sum_{i=1}^{N} \gamma_i \varphi(\|x-x_i\|) + \beta_1^i + \beta_2^i y + \beta_3^i x + \beta_4^i z
\end{align*}
\]

- Several different radial function (kernel) can be employed:

<table>
<thead>
<tr>
<th>RBF</th>
<th>( \varphi(r) )</th>
<th>RBF</th>
<th>( \varphi(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spline type (Rn)</td>
<td>( r^n, n \text{ odd} )</td>
<td>Inverse multiquadratic (IMQ)</td>
<td>( \frac{1}{\sqrt{1 + r^2}} )</td>
</tr>
<tr>
<td>Thin plate spline</td>
<td>( r^n \log(r), n \text{ even} )</td>
<td>Inverse quadratic (IQ)</td>
<td>( \frac{1}{1 + r^2} )</td>
</tr>
<tr>
<td>Multiquadratic (MQ)</td>
<td>( \sqrt{1 + r^2} )</td>
<td>Gaussian (GS)</td>
<td>( e^{-r^2} )</td>
</tr>
</tbody>
</table>
BGM Background

• **BGM** approach is based on the observation that **biological** structures growth is driven by **local** level of **stress**.

• Bones and trees’ trunks are able to **adapt the shape** to mitigate the stress level due to external loads.

• The process is driven by stress **value at surfaces**. Material can be **added or removed** according to local values.

• Was proposed by Mattheck & Burkhardt in 1990*

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The BGM idea is that surface growth can be expressed as a linear law with respect to a given threshold value:
\[ \dot{\varepsilon} = k \left( \sigma_{Mises} - \sigma_{ref} \right) \]

Waldman and Heller* refined this first approach proposing a multi peak one:
\[ d_i^j = \left( \frac{\sigma_i^j - \sigma_{th}^i}{\sigma_{th}^i} \right) \cdot s \cdot c, \quad \sigma_{th}^i = \max(\sigma_i^j) \text{ if } \sigma_i^j > 0 \quad \text{or} \quad \sigma_{th}^i = \min(\sigma_i^j) \text{ if } \sigma_i^j < 0 \]

In RBF Morph ANSYS Workbench ACT extension a different implementation is present and different stress types can be used to modify the surface shape:
\[ S_{node} = \frac{\sigma_{node} - \sigma_{th}}{\sigma_{max} - \sigma_{min}} \cdot d \]

<table>
<thead>
<tr>
<th>Stress/strain type</th>
<th>Equation</th>
<th>Stress/strain type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>von Mises stress</td>
<td>( \sigma_e = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} )</td>
<td>Stress intensity</td>
<td>( \sigma_e = \max(</td>
</tr>
<tr>
<td>Maximum principal stress</td>
<td>( \sigma_e = \max(\sigma_1, \sigma_2, \sigma_3) )</td>
<td>Maximum Shear stress</td>
<td>( \sigma_e = 0.5 \cdot (\max(\sigma_1, \sigma_2, \sigma_3) - \min(\sigma_1, \sigma_2, \sigma_3)) )</td>
</tr>
<tr>
<td>Minimum principal stress</td>
<td>( \sigma_e = \min(\sigma_1, \sigma_2, \sigma_3) )</td>
<td>Eqv. plastic strain</td>
<td>( \varepsilon_e = \left[ 2(1 + \nu') \right]^{-1} \cdot \left( 0.5 \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2} \right) )</td>
</tr>
</tbody>
</table>

Challenges

• Currently the mesh morphing allows to obtain complex shape modifications without remeshing, but can require a lot of efforts in order to maintain specific manufacturing constraints.

• In several industrial application the capability of replicate the shape modification along a direction or around an axis is a strong requirement.

• RBF Morph ACT extension introduced in the last version a new feature in order to satisfy these requirements.

• The Coordinate Filtering feature allows the user to replicate a specific RBF solution (i.e. shape modification) along or around a specified axis.
Applications Description

• To demonstrate the effectiveness of the Coordinate Filtering feature two applications were developed: a first one to apply a linear manufacturing constraint and a second one to apply a circular manufacturing constraint.
Applications Description – linear manufacturing constraints

• The bracket was constrained at the hole and loaded at the upper surface.

• Von Mises stress hot spots are located at hole and at fillet. The latter one will be the target of the optimization.

• Maximum von Mises stress at fillet in baseline configuration is 156 MPa.
Applications Description – circular manufacturing constraints

• The pin was constrained at the lower surface and loaded at the upper surface by means of a remote force.

• Von Mises stress hot spots is located at fillet and will be the target of the optimization.

• Maximum von Mises stress at fillet in the baseline configuration is 132 MPa.
Results – linear manufacturing constraints – parameters

• Parameter based optimization was set up with 3 **parameters**. Shape resulting from points displacement was replicated using **Coordinate Filtering**.

• **Ansys Design Xplorer** was employed to optimize shape using the **Response Surface Optimization**:

<table>
<thead>
<tr>
<th>Design of Experiment type</th>
<th>Latin Hypercube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samples type</td>
<td>CCD Samples</td>
</tr>
<tr>
<td>Response Surface type</td>
<td>Kriging</td>
</tr>
<tr>
<td>Kernel type</td>
<td>Variable</td>
</tr>
<tr>
<td>Refinement points</td>
<td>3 – candidate points</td>
</tr>
</tbody>
</table>
Results – linear manufacturing constraints – parameters

• With the optimized configuration obtained by Response Surface Optimization, the maximum von Mises stress value is 122 MPa, reduced by 21.8%.

• The optimized shape is compliant with linear manufacturing constraint even if it was obtained controlling only 3 points.
Results – linear manufacturing constraints – BGM

- When using BGM final shape can be very complex.
- Coordinate Filtering is required if manufacturing constraints are required.

Not filtered BGM

Not filtered vs. filtered BGM – amplified displacements
Results – linear manufacturing constraints – BGM

- **BGM** optimization was performed on the fillet surfaces using as threshold *von Mises stress* **100 MPa** and *maximum displacement* **1 mm**. The BGM optimization was iterated **10 times**.

- With the optimized configuration obtained by BGM optimization, the maximum *von Mises stress* value is **108 MPa**, reduced by **30.7%**.
Results – circular manufacturing constraints

- The same **3-parameters** approach was applied to the pin model.
- In this case with the parameter based optimization the maximum von Mises stress in the fillet area is reduced to 95 MPa, a **reduction of 28%** with respect the baseline configuration.
Results – circular manufacturing constraints

- **BGM** optimization was performed on the fillet surfaces using as **threshold** von Mises stress **75 MPa** and **maximum displacement 5 mm**. The BGM optimization was iterated **10 times**.

- With the optimized configuration obtained by BGM optimization, the maximum von Mises stress value is **95 MPa**, **reduced by 28%**.
Conclusions

• A **methodology** to obtain **optimized shape** suitable for traditional manufacturing processes **was developed**.

• The methodology was developed using **Ansys Workbench** and the **RBF Morph ACT** extensions.

• Optimization was performed using **BGM** and **parametric optimization** which, generally speaking, do not guarantee that linear or cyclic symmetry are respected.

• It was demonstrated that with these tools the **linear and circular features** can be **preserved** in the optimized configuration.

• Optimization was performed directly **controlling the shape** (parameter based) and **exploiting numerical results** regarding surface stresses (BGM).

• With both approaches stress reduction was between the range of **21% - 30%**.

• Proposed methodology can be successfully **adopted** and **implemented** in the design cycle of parts or components that are subjected to circular and linear manufacturing constraints.
Thank You For Your Kind Attention!
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